THE BATHYSTROPHIC STORM SURGE MODEL

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INTRODUCTION

Many sophisticated mathematical models have been developed in recent years that provide accurate three-dimensional estimates of energy flux and flooding that can be caused by a passing hurricane. All numerical models, regardless of sophistication of methodology must use the Bathystrophic Storm Tide Theory to estimate the rise of water on the open coast taking into account the combined effects of direct onshore and along shore wind-stress components of the strom on the surface of the water, and the effects of the Coriolis force (bathystrophic effect), and the different pressure effects.

Mathematical models using the Bathystrophic Storm Tide Theory can be quasi-one-dimensional, two dimensional, or three-dimensional numerical schemes. The simplest, which is described here, is a quasi-one-dimensional model which is a steady-state integration of the wind stresses of the hurricane winds on the surface of the water from the edge of the Continental Shelf to the shore, taking into consideration some of the effects of bottom friction and the along shore flow caused by the earth's rotation.

The bathystrophic contribution to hurricane surge can be explained as follows: In the northern hemisphere hurricane winds approaching the coast have a counterclockwise motion. Because of the Coriolis effect, the flow of water induced by the cyclonic winds will deflect to the right, causing a rise in the water level. The bathystrophic storm tide, therefore, is important in producing maximum surge even when winds blow parallel to the coast. Coastal morphology may also affect the extent of rise of water. However, in this model the surge is calculated only along a single traverse line at a time over the Continental Shelf for a straight open-ocean coast. Thus, it is labeled as quasi-one-dimensional.

The numerical model approach summarized here, is based on the Bathystrophic Storm Tide Theory and used to estimate the rise of water on the open coast by taking into account the combined effects of direct onshore and along shore wind-stress components on the water surface and the effects of the Coriolis force. Such a simple model uses the onshore and along shore wind-stress components of a moving wind field over the Continental Shelf, and a frictional component of bottom stress. The nonlinear storm surge is computed at selected points along the traverse by integrating numerically the one-dimensional hydrodynamic equations of motion and continuity.

The hurricane surge estimated by this simple model is a composite of water elevation obtained from components of the astronomical tide, the atmospheric pressure, the initial rise, the rises due to wind and bottom friction stresses, and wave setup.

Obviously such a basic model has its limitations. A hurricane is not stationary, and as it moves towards the coast, the wind speeds may increase and wind vectors will change direction changing frictional effects on the water surface. Such changes cannot always be predicted with accuracy to introduce them into the model.

The prediction of storm surge resulting from the combined meteorologic, oceanic, and astronomic effects coincident with the arrival of a hurricane at the coast is important in warning the public and in the planning and the design of important coastal structures. Increasing requirements for large coastal installations, have required conservative criteria in obtaining estimates of potential storm surges from hurricanes.

The bathystrophic theory on which hurricane surge prediction is based, represents an approximation to the complete storm-generation process. Therefore, such a model of prediction is limited by a number of initial conditions and assumptions. In most instances, the bathystrophic approximation appears to give a reasonable estimate of the open-coast surge; however, at times the surge estimate can be in error by a factor of 2 or more. However, the accuracy of hurricane prediction can be improved through calibration with known historical data - something which was accomplished successfully by the author more than 25 years ago.

More recently developed numerical models using a three dimensional approach, faster and more efficient computers and more accurate weather data from satellites, have greater potential for more accurate prediction. However, the fundamental principles in the prediction of hurricane surge described here, remain essentially the same.

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II. THE BATHYSTROPHIC STORM SURGE MODEL

The numerical model described here is based on the Bathystrophic Storm Tide Theory and is used to estimate the rise of water on the open coast taking into account the combined effects of direct onshore and alongshore wind-stress components on the surface of the water, and the effects of the Coriolis force (bathystrophic effect), and the different pressure effects. This model can be described as a quasi-one-dimensional numerical scheme, which is a steady-state integration of the wind stresses of the hurricane winds on the surface of the water from the edge of the Continental Shelf to the shore, taking into consideration some of the effects of bottom friction and the alongshore flow caused by the earth's rotation. The bathystrophic contribution to hurricane surge can be explained as follows: In the northern hemisphere hurricane winds approaching the coast have a counterclockwise motion. Because of the Coriolis effect, the flow of water induced by the cyclonic winds will deflect to the right, causing a rise in the water level. The bathystrophic storm tide, therefore, is important in producing maximum surge even when winds blow parallel to the coast. Coastal morphology may also affect the extent of rise of water. However, in this model the surge is calculated only along a single traverse line at a time over the Continental Shelf for a straight open-ocean coast.

The model uses the onshore and alongshore wind-stress components of a moving wind field over the Continental Shelf, and a frictional component of bottom stress. The nonlinear storm surge is computed at selected points along the traverse by integrating numerically the one-dimensional hydrodynamic equations of motion and continuity.

The computed surge is a composite of water elevation obtained from components of the astronomical tide, the atmospheric pressure, the initial rise, the rises due to wind and bottom friction stresses, and wave setup.

1. Basic Assumptions, Conditions, and Limitations.

The bathystrophic theory on which this model is based, represents an approximation to the complete storm-generation process. Therefore, the model is limited by a number of initial conditions and assumptions. In most instances, the bathystrophic approximation appears to give a reasonable estimate of the open-coast surge; however, at times the surge estimate could be in error by a factor of 2 or more. The basic equations which govern the generation of storm surge will not be derived here, but to understand the bathystrophic approximation and its limitations, it is important to emphasize the assumptions, initial conditions, and the hydrodynamic processes neglected in development.

The following initial conditions are placed on the basic equations which govern storm surge generation (Bodine, 1971):

a. The hurricane creates a disturbance on the ocean surface of such horizontal dimensions so that $L \gg D$ and $L \ll R_E$, where L is the length of the disturbance; D is the depth of the water (at the edge of the Continental Shelf), and R_E is the radius of the earth. It is also assumed that:

(1) The space derivatives of the current velocity and acceleration can be neglected. Thus, the vertical pressure gradient is hydrostatic, and vertical accelerations are negligible.

(2) The curvature of the earth can be neglected and a flat earth approximation can be used.

b. The acceleration due to the earth's rotation is a constant.

c. Water density ρ is a constant, and internal forces due to viscosity can be neglected.

d. The seabed is fixed, impermeable, and forms the lower boundary.

e. Surface storm waves are linearly superimposed on storm surge. A basic assumption of the model is that the surge involves only horizontal fluid motions. In respect to wave theory, such horizontal fluid motions are often referred to as *long waves*. The water motion associated with the propagation of long waves is in a continuous state of gradual change.

2. Hydrodynamic Equations.

Integrations of the primary hydrodynamic equations describing the storm surge problem have been shown by Haurwitz (1951), Welander (1961), Fortak (1962), Platzman (1963), Reid (1964), and Harris (1967). These derivations show the actual approximations involved. In the bathystrophic model by Bodine (1971), the basic equations are the simplified and vertically integrated equations of continuity expressing conservation of mass and motion, according to Newton's second law. The equations were taken directly from Bodine in integrated form for the purpose of illustrating the principal approximations of the bathystrophic model.

The governing two-dimensional hydrodynamic equations in a volume-transport form, appropriate for tropical or extratropical storm surge problems on the open coast and in enclosed or semienclosed basins, are as follows:

$$\frac{\partial U}{\partial t} + \underbrace{\frac{\partial M_{xx}}{\partial x} + \frac{\partial M_{xy}}{\partial y}}_{\text{Advection of Momentum}} = \underbrace{+fV}_{\text{Coriolis}} \underbrace{- gD\frac{\partial S}{\partial x}}_{\text{Surface Slope}} + \underbrace{gD\frac{\partial \xi}{\partial x}}_{\text{Inverse Barometer}} + \underbrace{gD\frac{\partial \zeta}{\partial x}}_{\text{Astro. Tide Potential}} + \underbrace{\frac{\tau_{sx}}{\rho}}_{\text{Wind Stress}} - \underbrace{\frac{\tau_{bx}}{\rho}}_{\text{Bottom Stress}} + \underbrace{W_x P}_{\text{Rainfall Rate}} \quad (3)$$

$$\frac{\partial V}{\partial t} + \overbrace{\frac{\partial M_{yy}}{\partial y} + \frac{\partial M_{xy}}{\partial x}} = \overbrace{-fU} \overbrace{- gD\frac{\partial S}{\partial y}} + \overbrace{gD\frac{\partial \xi}{\partial y}} + \overbrace{gD\frac{\partial \zeta}{\partial y}} + \overbrace{\frac{\tau_{sy}}{\rho}} - \overbrace{\frac{\tau_{by}}{\rho}} + \overbrace{W_y P} \quad (4)$$

$$\frac{\partial S}{\partial t} + \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = P. \quad (5)$$

where M_{xx}, M_{yy}, M_{xy}, U, and V are given by:

$$M_{xx} = \int_{-d}^{S} u^2\,dz; \quad M_{yy} = \int_{-d}^{S} v^2\,dz; \quad M_{xy} = \int_{-d}^{S} uv\,dz,$$

$$U = \int_{-d}^{S} u\,dz; \qquad V = \int_{-d}^{S} v\,dz\,.$$

where,

U, V = x and y components, respectively, of volume transport per unit width,

t = time,

M_{xx}, M_{yy}, M_{xy} = momentum transport quantities,

f = $2\omega \sin\phi$ = Coriolis parameter,

ω = angular velocity of earth = $2\pi/24$ radians per hour (7.29×10^5 radians per second),

ϕ = geographical latitude,

τ_{sx}, τ_{sy} = x and y components of surface wind stress,

τ_{bx}, τ_{by} = x and y components of bottom stress,

ρ = mass density of water,

W_x, W_y = x and y components of windspeed,

ξ = atmospheric pressure deficit in head of water,

ζ = astronomical tide potential in head of water,

u, v = x and y components, respectively, of current velocity,

p = precipitation rate (depth/time),

S = setup of water surface above SWL,

g = gravitational acceleration,

D = depth of water at edge of Continental Shelf,

d = depth of water on the Continental Shelf,

and

θ = angle of wind measured counterclockwise from the x-axis.

Equations (3) and (4) are the equations of motion, while equation (5) is the continuity relation for a fluid of constant density. These basic equations provide a complete description of nearly all horizontal water motions resulting from hurricane surge. An exact solution of these equations would be very desirable for solving the surge problem; however, it is difficult to obtain. The model described here obtains only a useful approximation by ignoring some terms in the basic equations. Accurate solutions can only be acquired by retaining the full two-dimensional characteristics of the surge problem.

3. The Bathystrophic Model Approximation.

Application of the Bathystrophic Storm Tide Theory of Freeman, Baer, and Jung (1957) to the hydrodynamic equations of motion and continuity for the solution of the surge problem requires a number of assumptions. According to Bodine (1971) these assumptions imply that: (a) there is no volume transport normal to the shore, (b) the onshore wind setup responds instantaneously to the onshore wind stress, (c) advection of momentum (field acceleration) is negligible, (d) the alongshore sea surface height is uniform, and (e) precipitation can be neglected. When applied to the terms of the equations of motion and continuity (1), (2), and (3), according to Bodine these assumptions have the following physical significance:

$$\frac{\partial U}{\partial t},\ fU,\ \frac{\partial U}{\partial x},\ \frac{\tau_{bx}}{\rho} \rightarrow 0, \text{ (no onshore water volume transport)},$$

$$M_{xx},\ M_{yy},\ M_{xy} \rightarrow 0, \text{ (the advection of momentum can be neglected)},$$

$$\frac{\partial S}{\partial y},\ \frac{\partial V}{\partial y} \rightarrow 0, \text{ (the alongshore sea surface and current are uniform)},$$

$$P \rightarrow 0, \text{ (the precipitation can be neglected)},$$

$\frac{\partial \xi}{\partial x}, \frac{\partial \xi}{\partial y} \rightarrow$ (The barometric effects are neglected in this approximation, but are accounted for elsewhere. These effects are discussed later.)

$\frac{\partial \zeta}{\partial x}, \frac{\partial \zeta}{\partial y} \rightarrow$ (The astronomical tide effects are neglected in this approximation but are accounted for in the final estimate of the surge on the coast. These effects are discussed later.)

Based on these assumptions, the motion equations (3) and (4), reduce to the following approximations:

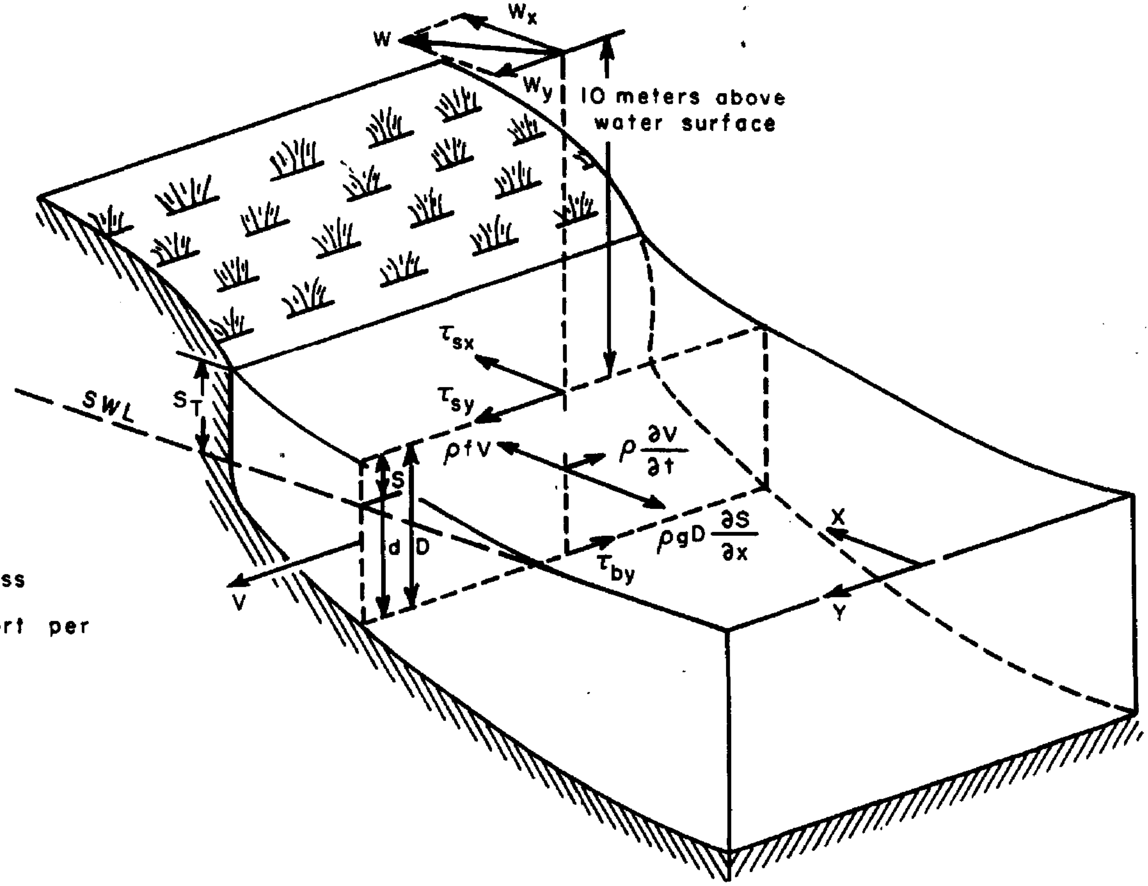
$$gD \frac{\partial S}{\partial x} = fV + \frac{\tau_{sx}}{\rho} , \tag{6}$$

$$\frac{\partial V}{\partial t} = \frac{\tau_{sy} - \tau_{by}}{\rho} . \tag{7}$$

The mass-continuity equation (5) is disregarded in the bathystrophic approximation because the assumptions make it unnecessary for a unique solution, and of no interest, for each term has been set equal to zero. The reduced equations (6) and (7) are now quasi-two-dimensional, since their solution can be obtained only along a single axis, the x-axis; however, the y-axis bathystrophic component of transport is retained and can be accounted for. The forces and responses for the bathystrophic approximation are shown in Figure 1. The weakness of the bathystrophic surge model lies on the numerous approximations outlined here. A reduction in the number of these approximations and the solution of the hydrodynamic equations in a more complete form may result in better estimates of storm surges.

4. Wind and Bottom Frictional Stresses.

Equations (6) and (7) include the x- and y-components of wind stress on the surface of the water, τ_{sx} and τ_{sy}, and the y component of bottom stress due to the water motion, τ_{by}. These are frictional stresses which need to be quantified for the solution of the surge problem. Formulas derived from theoretical solutions and physical experiments have been introduced which provide reasonable values of frictional forces at the bottom and water surface boundaries. However, these stress values have not been verified for the entire spectrum of conditions encountered in nature. Furthermore, the surface wind stresses and the bottom stresses must be "coupled" for the numerical computation of hurricane surge described in this report. Although surface and bottom stresses can be obtained individually from empirical experiments, the combined interaction of these stress factors does not obey a linear relationship. Friction models which take into account the interaction of surface wind and bottom friction stresses have been proposed by Reid (1957), Platzman (1963), and Jelesnianski (1967, 1970). These proposals, however, fail to describe the interactions of stresses in shallow water near the shore. Similarly, extrapolation of the surface wind stress relationship, as determined from lower windspeeds, to extreme wind conditions may not be realistic, as there may be interaction of other unknown or unmeasurable variables. Nonetheless, reasonable estimates of boundary stresses can be obtained if simplified stress laws are used, and the influence of vertical velocity distribution is neglected. Assuming horizontally uniform flows of wind and water at the water surface and seabed boundaries, NOTE: Various scales have been distorted to give a clearer pictorial representation.



LEGEND:

- SWL = Stillwater Level
- S_T = Total Setup at Shore
- S = Setup
- d = depth below SWL
- D = Total depth
- τ_{sx}, τ_{sy} = x,y components of wind stress
- V = y-component of water transport per unit width (of x)
- W = Windspeed
- W_x, W_y = x,y components of windspeed
- τ_{by} = y-component of bottom stress
- f = Coriolis parameter
- ρ = density of water
- g = gravity
- t = time

Figure 1. Schematic of forces and responses for bathystrophic approximation (U. S. Army, Corps of Engineers, Coastal Engineering Research Center, 1973).

respectively, similar formulas can be adopted for representing the wind and bottom friction stresses. Considering these flows, the mean shear stress at the boundary, τ relates to the mean velocity gradient $\bar{v}$ as follows:

$$\tau = \gamma\rho\bar{v}^2,$$

where ρ is the water density, and γ is a dimensionless resistance coefficient. To retain the proper sign consistent with the coordinate system used, the above equation can be written as follows:

$$\tau = \gamma\rho\bar{v}|\bar{v}|. \tag{8}$$

Accordingly, the shear stress at the bottom (τ_{by}) divided by water density, consistent with the stress term required for equation (7), is:

$$\frac{\tau_{by}}{\rho} = Kv|v|, \tag{9}$$

where the bed friction coefficient K, and the y-component of the water velocity, v, replace γ and $\bar{v}$ respectively, in equation (8). The bed friction coefficient K, as presented here, is dimensionless in accordance to the Prandtl-von Karman Boundary-Layer Theory (Prandtl, 1935; von Karman, 1930). In the bathystrophic model used in this study (Bodine, 1971), the Prandtl-von Karman Boundary-Layer Theory was chosen because of the simplicity in computation. In transport form, equation (9) is given by Bodine as:

$$\frac{\tau_{by}}{\rho} = \frac{KV|V|}{D^2} \text{ (bottom shear stress).} \tag{10}$$

For typical seabed conditions, the bottom friction coefficient K has been assigned values ranging from 2×10^{-3} and 5×10^{-3}. K is related to the coefficient of Chezy C and the Darcy-Weisbach friction factor f_f as follows:

$$K = \frac{g}{C^2} = \frac{f_f}{2}.$$

The wind-induced water surface stress, in accordance with equation (8), is given by:

$$\tau_s = \rho k W^2 = \rho k W|W|, \tag{11}$$

where W is the wind velocity as given at standard anemometer level (30 feet above the wave surface, based on 10-minute averages), and k is a dimensionless surface friction coefficient. The square of the wind velocity is given as an absolute term W|W| rather than W^2 to retain the proper sign consistent with the coordinate system used. According to a relationship worked out by Van Dorn (1953), the wind-stress coefficient is a function of the windspeed given by:

$$k = K_1 + K_2 \left(1 - \frac{W_c}{W}\right)^2 \quad \text{for } W \geqslant W_c , \tag{12}$$

where the constants K_1 and K_2 were derived empirically by Van Dorn to be 1.1×10^{-6} and 2.5×10^{-6} respectively, and W_c is a critical windspeed taken as 14 knots (about 16 miles per hour). When $W \leqslant W_c$, equation (12), reduces to:

$$k = K_1 . \tag{13}$$

On the basis of equation (11), the x- and y-components of wind shear stress can be written as:

$$\left.\begin{aligned} \frac{\tau_{sx}}{\rho} &= kW^2 \cos\theta \\ \frac{\tau_{sy}}{\rho} &= kW^2 \sin\theta , \end{aligned}\right\} \quad \text{(wind shear stress)} \tag{14}$$

where θ is the angle between the x-axis and the local wind vector.

Equations (11) through (14) can now be introduced into the reduced equations of motion of the bathystrophic approximation; equations (6) and (7), can now be written as follows:

$$\frac{\partial S}{\partial x} = \frac{1}{gD}\left[fV + kW^2 \cos\theta\right], \tag{15}$$

$$\frac{\partial V}{\partial t} = kW^2 \sin\theta - \frac{KV^2}{D^2} \tag{16}$$

These equations are simplified forms of the hydrodynamic equations which are applied to the estimation of storm surge. The solution of these equations by numerical integration is given later in this report.

5. Numerical Solution of the Surge Problem.

A discussion of the numerical scheme for computing bathystrophic storm surge is given by Bodine (1971). The basic concepts are detailed here to show the geometry of the finite-difference numerical method and solution of the numerical equations of setup and flux.

Equations (15) and (16) giving setup and flux, respectively, can be solved in finite increments of time and space, assuming the functional relationships of the bathystrophic equations as continuous over the entire computing interval. The two-dimensional finite step method of the numerical scheme is illustrated in Figures 2 and 3 for discrete increments of space, $(\Delta_x)_i$ and time $(\Delta_t)_n$ along a single Cartesian axis, the computational plane represented by the traverse. For convenience, the seabed slope AB is shown to be uniform (Fig. 3). Point C represents position of the edge of the Continental Shelf or the most seaward point along the traverse, above point A. The line CB represents the equilibrium condition of the sea surface before being affected by the approaching hurricane winds. The line CD represents the sea surface altered by the cumulative effects of the winds, astronomical tide, initial rise, pressure differences and storm waves. A detailed explanation of the variables contributing to storm surge on the shore is discussed later. The cumulative water elevation (S) associated with the hurricane is DB, while point B represents the shore intercept of the traverse.

For convenience, a boundary condition is placed on the model at point B where the shoreline is represented by a vertical wall and surge is calculated at a one-half step increment in front of this wall. The model treats the final water elevation as a static condition, and the shoreline as a vertical impermeable boundary. However, in reality, the shore is a sloping surface and dynamic processes and momentum forces associated with water transport and storm waves may result in greater surge elevation along the coast.

An initial assumption of the numerical system is that, at the beginning of the calculation, when $t = t_o$, the system is in an equilibrium state, with a uniform water surface, and no currents. This implies that the water flux, V, is zero at $t = t_o$ and that S has a constant value for the system. Although in reality the system does not exist in a state of complete equilibrium, it is a reasonable assumption for the calculation. Later, this assumption is of little consequence, since the response of the system reflects only the effects of the input-forcing functions.

The discrete position x along the traverse line at any time level, t, is defined as:

$$x = x_o - \sum_{i=1}^{IM} (\Delta x)_i \ , \tag{17}$$

and the time level t is given by:

$$t = t_o + \sum_{n=1}^{NM} (\Delta t)_n \ , \tag{18}$$

where x_o as shown in Figure 3, is the distance from the shore intercept of the traverse to the most seaward point of the traverse. The summation of Δx is for all i intervals up to and including IM, so that at the shore $x = x_o$. Although x, according to the coordinate system shown here, would be negative, changing the sign of x to a positive value does not

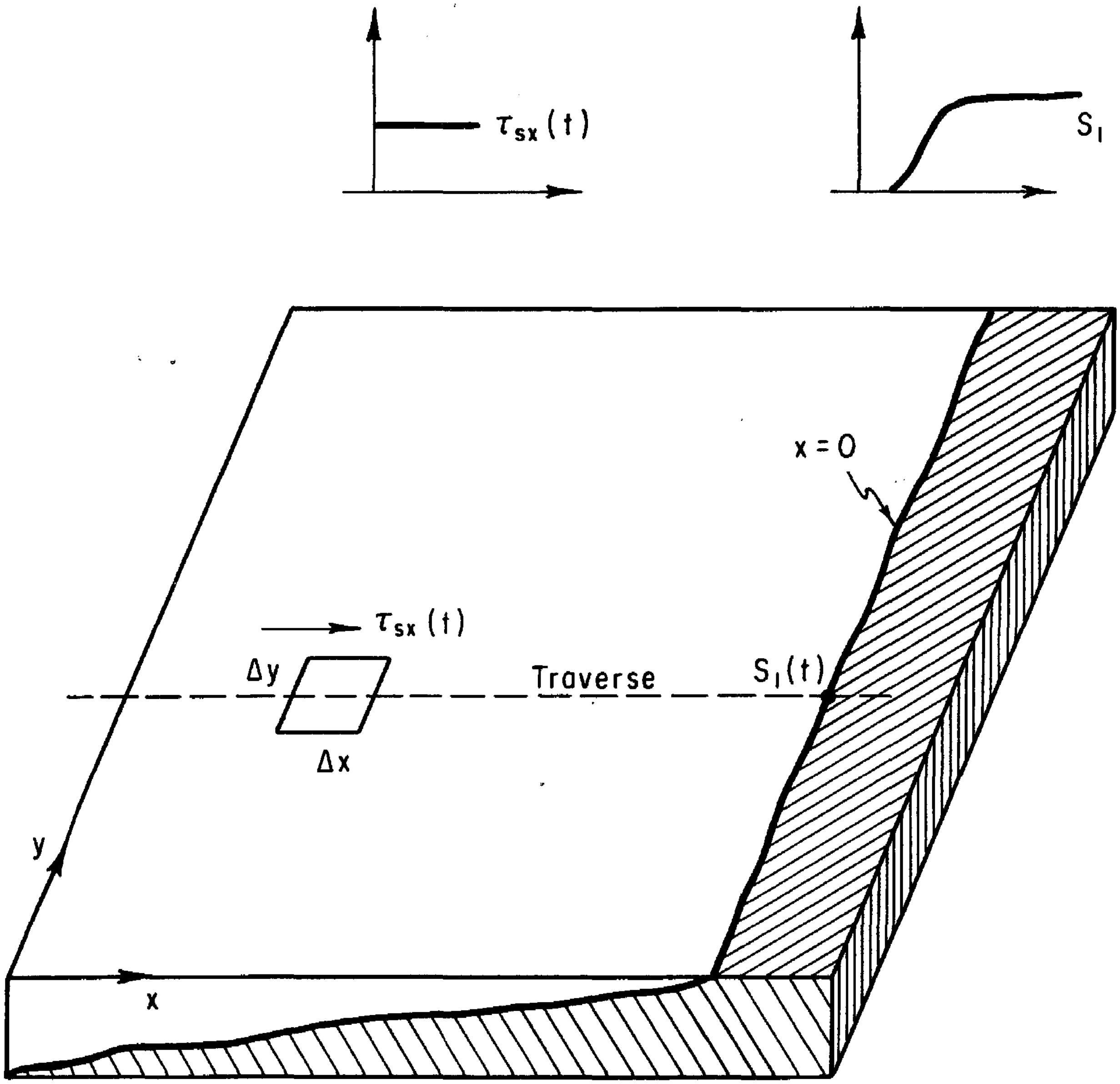


Figure 2. The response of sea level $S_1 \Delta x \Delta y$ due to a step function in the wind stress τ_{sx} acting over a unit area $\Delta x \Delta y$.

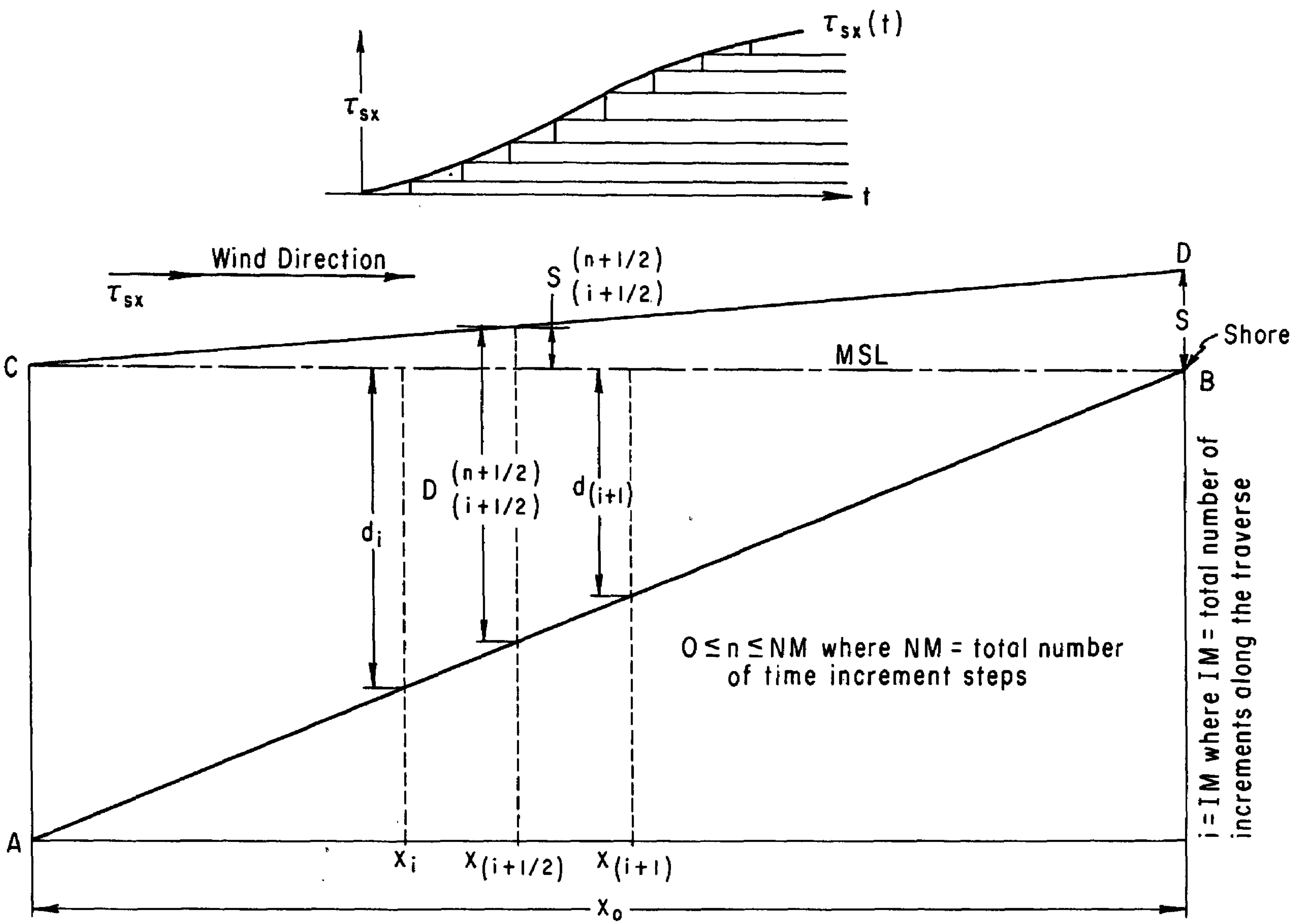


Figure 3. Numerical solution by summation of step functions.

change the computational method. The time scale at the beginning of the calculation at the seaward point of the computational line is t_o and the time level is the summation of Δ_t for all n's up to, and including a specified value n = NM.

From the chosen geometry of the present numerical scheme, the values of wind stress τ_s, seabed depths below the undisturbed level d, and the Coriolis effects are supplied at all discrete positions of i. The setup S, the total water depth at each increment, and the value of volume transport V, are evaluated at intervals, i + ½.

The total water depth $\left(D^{n+\frac{1}{2}}_{i+\frac{1}{2}}\right)$ midway between two time levels n and n + 1 is centered between the points x_i and x_{i+1}, and the cumulative water depth is given by:

$$D^{n+\frac{1}{2}}_{i+\frac{1}{2}} = \frac{d_i + d_{i+1}}{2} + S_e + \frac{S^n_A + S^{n+1}_A}{2} + \left(S_x + S_y\right)^n_{i+\frac{1}{2}}$$

$$+ \frac{1}{4}\left(\left[\left(S_{\Delta p}\right)_i + \left(S_{\Delta p}\right)_{i+1}\right]^n + \left[\left(S_{\Delta p}\right)_i + \left(S_{\Delta p}\right)_{i+1}\right]^{n+1}\right), \qquad (19)$$

where

S_e = initial rise in the water level at the time the storm surge computations are started,

S_A = setup due to astronomical tide,

$S_{\Delta p}$ = atmospheric pressure setup in feet, given by:

$$S_{\Delta p} = 1.14 \left(p_n - p_o\right) \left(1 - e^{-R/r}\right),$$

which is an approximate relationship when pressure is expressed in inches of mercury and where p_n is the pressure at the periphery of the storm, and r is the radial distance from the storm center to the computation point on the traverse line and S_x and S_y are the components of the storm setup given by:

$$S_x = \sum_{j=1} \left(\Delta S_x\right)_j, \qquad (20)$$

$$S_y = \sum_{j=1} \left(\Delta S_y\right)_j. \qquad (21)$$

The physical significance of equations (20) and 21) is that total wind setup for any discrete position along the traverse is the setup in that reach superimposed cumulatively on the setups in all reaches seaward.

For a new time level, n + 1, the total water depth is given by:

$$D^{n+1}_{i+½} = \frac{d_i + d_{i+1}}{2} + S_e + S_A^{n+1} + \left(S_x + S_y\right)^n_{i+½}$$

$$+ \frac{1}{2} \left[\left(S_{\Delta p}\right)_i + \left(S_{\Delta p}\right)_{i+1}\right]^{n+1} . \tag{22}$$

From Figure 3 and equations (19) and (22) it is apparent that a small error is introduced each time D is calculated because the term $(S_x + S_y)$ is taken at the previous time level rather than at time (n + ½) for equation (19) and (n + 1) for equation (22). The reason is that the correct values are not known for these time intervals; therefore, an approximation is made. This error, however, is minimized by using small increments of time and space in the calculations.

The differential hydrodynamic equations (15) and (16) can now be solved by numerical integration. Equation (15) gives the cumulative setup resulting from onshore and alongshore effects. The onshore, wind-induced component, according to Bodine (1971) can be separated from the alongshore bathystrophic component and equation (15) can be written in its equivalent forms as follows:

$$\frac{\partial S_x}{\partial x} = \frac{kW^2 \cos\theta}{gD} , \tag{23}$$

$$\frac{\partial S_y}{\partial x} = \frac{fV}{gD} , \tag{24}$$

where the total setup along the x-axis is the sum of the two, given by

$$\frac{\partial S}{\partial x} = \frac{\partial S_x}{\partial x} + \frac{\partial S_y}{\partial x} .$$

Numerical integration of equations (23) and (24) will give the following numerical analogs (Bodine, 1971):

$$\left(\Delta S_x\right)^{n+1}_{i+½} = \frac{\Delta x}{2gD^{n+1}_{i+½}} \left(A_i + A_{i+1}\right)^{n+1} , \tag{25}$$

$$\left(\Delta S_y\right)^{n+1}_{i+½} = \frac{\Delta x}{2gD^{n+1}_{i+½}} \left(f_i + f_{i+1}, \; V^{n+1}_{i+½} , \tag{26}$$

where A is a Kinematic form of wind stress given by:

$$A = kW^2 \cos\theta .$$

Similarly, the alongshore differential equation (16) of water flux can be resolved by numerical integration as:

$$\frac{\partial V}{\partial t} = kW^2 \sin\theta - \frac{KV^2}{D^2},$$

or

$$\frac{\partial V}{\partial t} = kW^2 \sin\theta - KV|V|D^{-2}.$$

However,

$$\frac{\partial V}{\partial t} \simeq \frac{V^{n+1}_{i+½} - V^{n}_{i+½}}{\Delta t}. \qquad (27)$$

The first term of equation (16), $kW^2 \sin\theta$ can be made equal to B, which is also a Kinematic form of the wind stress, so that,

$$B = kW^2 \sin\theta \simeq \frac{1}{4}\left[\left(B_i + B_{i+1}\right)^n + \left(B_i + B_{i+1}\right)^{n+1}\right]. \qquad (28)$$

The second term of equation (16) can be approximated as follows:

$$KV|V|D^{-2} \simeq KV^{n+1}_{i+½}\left|V^{n}_{i+½}\right|\left(D^{-2}\right)^{n+½}_{i+½}. \qquad (29)$$

Substituting equations (27), (28), and (29) into equation (16), yields:

$$\frac{V^{n+1}_{i+½} - V^{n}_{i+½}}{\Delta t} = \frac{1}{4}\left[\left(B_i + B_{i+1}\right)^n + \left(B_i + B_{i+1}\right)^{n+1}\right]$$

$$- KV^{n+1}_{i+½}\left|V^{n}_{i+½}\right|\left(D^{-2}\right)^{n+½}_{i+½}. \qquad (30)$$

Multiplying equation (30) by Δ_t and transposing the term $V^{n}_{i+½}$ yields:

$$V^{n+1}_{i+½} = \frac{1}{4}\left[\left(B_i + B_{i+1}\right)^n + \left(B_i + B_{i+1}\right)^{n+1}\right]\Delta t$$

$$- KV^{n+1}_{i+½}\left|V^{n}_{i+½}\right|\left(D^{-2}\right)^{n+½}_{i+½}\Delta t + V^{n}_{i+½}, \qquad (31)$$

transposing the term

$$KV_{i+½}^{n+1} \left| V_{i+½}^{n} \right| \left(D^{-2}\right)_{i+½}^{n+½} \Delta t,$$

yields:

$$V_{i+½}^{n+1} + KV_{i+½}^{n+1} \left| V_{i+½}^{n} \right| \left(D^{-2}\right)_{i+½}^{n+½} \Delta t$$

$$= V_{i+½}^{n} + \frac{1}{4} \left[\left(B_i + B_{i+1}\right)^{n} + \left(B_i + B_{i+1}\right)^{n+1}\right] \Delta t. \qquad (32)$$

Factoring out the term $V_{i+½}^{n+1}$ and dividing equation (32) by the term

$$1 + K \left| V_{i+½}^{n} \right| \left(D^{-2}\right)_{i+½}^{n+½} \Delta t,$$

becomes:

$$V_{i+½}^{n+1} = \frac{\left(\frac{1}{4}\right)\left[\left(B_i + B_{i+1}\right)^{n} + \left(B_i + B_{i+1}\right)^{n+1}\right] \Delta t + V_{i+½}^{n}}{1 + K\left| V_{i+½}^{n} \right| \Delta t \left(D^{-2}\right)_{i+½}^{n+½}}, \qquad (33)$$

which is the numerical analog of equation (16).

In the numerical analogs of the bathystrophic equations, nonuniform spacing Δx and time, Δt, steps can be taken. This permits coarse spacing, Δx, where the seabed is relatively flat, and fine spacing near the shore where the bed slope changes rapidly. Similarly, nonuniform time steps, Δt, permit more frequent storm-surge computations during the period when rapid water level changes are anticipated.

Based on this numerical scheme and logic, calculations of surge are started at the seaward boundary to the shore-intercept through all prescribed spatial positions on the traverse at the initial time level. The process is repeated for each prescribed time level, and continued for the entire temporal range. For each discrete position along the traverse line, the flux V, at each new time level, is evaluated based on the flux V at the previous time level. Similarly the stress term B, and depth D, are evaluated as the average values in the incremental domain of $(x + \Delta x)$ and $(t + \Delta t)$. Determination of V at each new time level can be made with equation (33), then using this value, determination of the x- and y-components of setup, ΔS_x and ΔS_y, can be made with equation (25) and (26), to obtain the total setups S_x and S_y.

As mentioned earlier, the total water level rise on the shore will be the summation of a number of components from the meteorological storm plus those unrelated to the storm. The total setup is given by:

$$S_T = S_x + S_y + S_{\Delta p} + S_e + S_A + S_W + S_L. \qquad (34)$$

Definitions of most components have been given. Some other components are related to the storm but some unrelated terms are provided as input to the surge computation. In equation (34), S_w is the wave setup at the shore due to breaking waves given (U.S. Army, Corps of Engineers, Coastal Engineering Research Center, 1973):

$$S_W = 0.19 \left[1 - 2.82 \left(\frac{H_b}{gT^2} \right)^{1/2} \right] H_b \, , \tag{35}$$

where H_b is the height of the breaking wave, g is the gravitational acceleration, and T is the wave period. The local setup or setdown S_L, is the deviation of the water surface from the computed water level due to such local effects as inland runoff inside the coastal barrier or the coastal hydrography. This component can only be estimated from full consideration of the influences of topography and hydrography not considered in the numerical computations. A schematic representation of the different setup components contributing to storm surge on the shore is shown in Figure 4.

Calculation of volume transport V, is based on repeated computations using the same formula, and can result in round-off errors, as each computed value will influence the values which remain to be determined. To ensure that the value of V does not exceed the maximum possible value, Bodine (1971) derived the following relationship. In an incremental form, equation (16) can be written as:

$$\Delta V = kW^2 \sin\theta \, \Delta t - KV^2 D^{-2} \, \Delta t \, ,$$

or

$$KV^2 D^{-2} \, \Delta t = kW^2 \sin\theta \, \Delta t - \Delta V \, .$$

For small

$$\Delta V, \; KV^2 D^{-2} \leqslant kW^2 \sin\theta \, .$$

Thus, the y-component of volume transport becomes

$$V \leqslant \sqrt{\frac{D^2 kW^2 \sin\theta}{K}} \, .$$

At the new time level, the above equation can be written in its numerical analog form, as:

$$V_{i+1/2}^{n+1} \leqslant \sqrt{\frac{\left| \left(B_i + B_{i+1} \right)^{n+1} \right| \left(D^2 \right)_{i+1/2}^{n+1}}{2K}} \, . \tag{36}$$

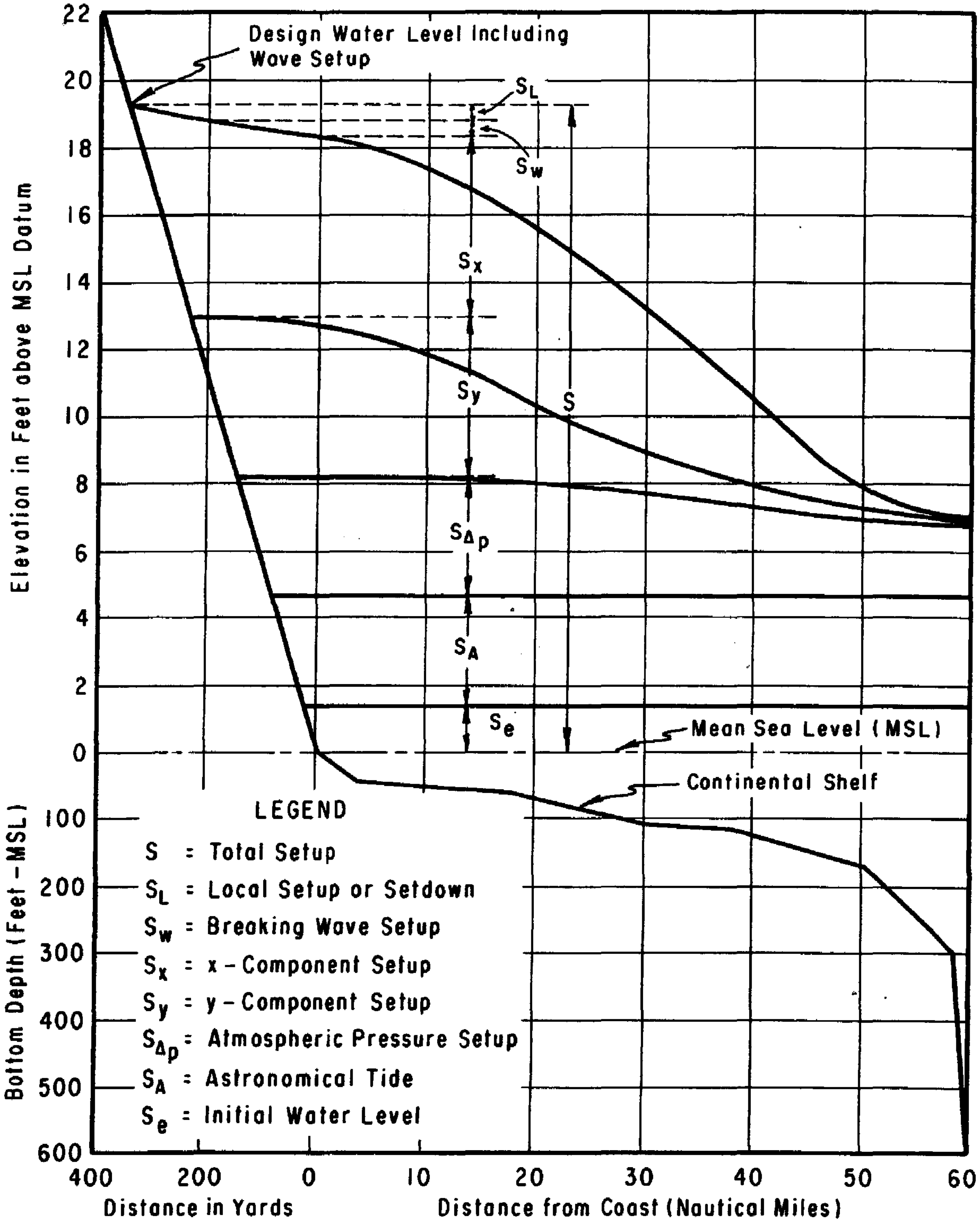


Figure 4. Various setup components over the Continental Shelf contributing to storm surge on the shore (Bodine, 1971).

According to this equation the absolute value of the flux must never exceed the term on the right-hand side. This relation is used as a check, and if this value is exceeded, the flux at the new time level, as an estimate, can be set equal to the value given by the right-hand side of equation (36). The integrated equations (25), (26), (33) and (36) can be used for the numerical solution of the surge problem. However, because of the inconsistency of units in the different terms used in these equations, it is desirable to absorb the invariant coefficients of these terms and substitute with constants. Such substitution reduces the possibility of errors and the equations are easier to use in the program. For example, weather charts usually present the wind data in knots. Distances taken from hydrographic maps are given in nautical miles, and depths in fathoms or feet. To eliminate conversion of units, Table 1 gives the dimensions of the variables used in the numerical scheme in four systems of units and the corresponding value of the constants for each system. The first column of units is given in the metric system while the other three are given in mixed units of the English system.

Table 1. Systems of Units for Storm Surge Computations

Parameters	Units and Constant Values			
	Metric	Mixed English		
Δx	km	nm	nm	mi
$\Delta S_x, \Delta S_y$	m	ft	ft	ft
g	m/sec^2	ft/sec^2	ft/sec^2	ft/sec^2
D	m	ft	ft	ft
A, B	(km/hr)^2	(nm/hr)^2	(mi/hr)^2	(mi/hr)^2
V	km^2/hr	nm^2/hr	mi^2/hr	mi^2/hr
f	hr^{-1}	hr^{-1}	hr^{-1}	hr^{-1}
Δt	hr	hr	hr	hr
C_1	3.94	269	203	176
C_2	2.06	141	106	92
C_3	(1,000)^2	(6,080)^2	(5,280)^2	(5,280)^2

(U.S. Army, Corps of Engineers, Coastal Engineering Research Center, 1973).

Equations (25), (26), (33), and (36) can now be given in a more simplified computational form, as follows:

$$\left(\Delta S_x\right)_{i+½}^{n+1} = \frac{C_1\ \Delta x}{D_{i+½}^{n+1}}\ \left(A_i + A_{i+1}\right)^{n+1}, \tag{37}$$

$$\left(\Delta S_y\right)_{i+½}^{n+1} = \frac{C_2\ \Delta x}{D_{i+½}^{n+1}}\ \left[\left(\sin\phi\right)_i + \left(\sin\phi\right)_{i+1}\right]\ V_{i+½}^{n+1}, \tag{38}$$

$$V_{i+½}^{n+1} = \frac{\left(\frac{1}{4}\right)\left[\left(B_i + B_{i+1}\right)^n + \left(B_i + B_{i+1}\right)^{n+1}\right]\ \left(\Delta t\right) + V_{i+½}^{n}}{1 + C_3\ \left|V_{i+½}^{n}\right|\Delta t\ \ K\left(D^{-2}\right)_{i+½}^{n+½}}, \tag{39}$$

$$V_{i+½}^{n+1} \leqslant \sqrt{\frac{\left|\left(B_i + B_{i+1}\right)^{n+1}\right|\ \left(D^2\right)_{i+½}^{n+1}}{2C_3 K}} \tag{40}$$

The values of the dimensional constants C_1, C_2 and C_3 in equations (37), (38), (39), and (40) will depend on the system of units used in performing the computations. In the present model, the English system of units is used, so $C_1=203$, $C_2=106$, and $C_3=(5280)^2$.

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